Location Choice and Quality Competition in Mixed Hospital Markets *

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Abstract

Many countries have opened up their health care markets for private for-profit providers to promote quality and choice for patients. Prices are regulated and providers compete in location and quality. We show that opening up a public hospital market typically raises quality, but that the private provider strategically locates towards the corner of the market to avoid costly quality competition. Social welfare depends on the size of the regulator's budget is and on altruism of the public provider. If the budget is large, high quality results and welfare is highest in duopoly whenever entry pays off. If the budget is small, quality levels in duopoly coincide with the quality level in monopoly. It can be optimal for the regulator not to use the full budget.

JEL codes: D43, I18, L13, L51

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1 Introduction

In recent years, governments in the Nordic countries have opened up the health care market for private for-profit providers. This is a market type where prices are regulated and competition is on location and quality. It is believed that competition brings forward higher quality and more choice for patients.

In Norway for example, the health care market for psychiatric care and substance abuse treatment was opened up for private for-profit providers in November 2015. Given that a private for-profit provider has the required licence to operate, he can now enter the market and start competing for patients. He is paid fixed prices per treatment determined by the government. In 2016 the government granted a budget of €41 million to finance the reform.[1]

In this paper, we examine how increased competition, elicited by opening up a public hospital market for a private hospital, impacts on hospital quality, on the location choice of the private hospital, and on social welfare. To address these issues we consider a mixed duopoly model with product differentiation. Our model has three stages and three active players: the regulator, the public hospital and the private hospital. At stage 1, the regulator determines the reimbursement given to the providers. At stage 2, the private provider locates on the Hotelling line. The public provider is already located in the market when the private provider decides whether to enter the market or not. Finally, at stage 3, the hospitals choose non-verifiable quality levels, and patients decide where to receive care. We assume that the cost structure of the two hospitals is similar, but that the public provider exhibits semi-altruistic preferences. Accordingly, (the health care workers in) the public hospital will trade off patient benefits against lower profit (see e.g. McGuire, 2000). However, the public hospital has to fulfill a non-negative-profit constraint.

The market analysis shows that the public hospital responds to the increased competition by raising it's quality level in most cases. When the price offered by the regulator is relatively high, quality will be raised to such a high level that the public hospital's zero-profit constraint is binding. Furthermore, the zero-profit constraint effectively dampens quality competition so that the private provider enters and locates in the interior of the Hotelling street. He obtains a market share of 1/6 of the market. On the other hand, if the regulated price is relatively low, the public provider would respond to the private provider's choice of locating closer to the middle of the market by raising quality. Since quality competition is costly, the private hospital locates itself towards the corner of the market (to dampen quality competition). In some cases, it will receive a market share of 1/3**.** If the regulated price is sufficiently low, the private hospital will enter the market, but both hospitals implement the minimum quality level. In these cases the private hospital avoids costly quality competition by locating towards the corner of the market.

When it comes to social welfare, the effect depends on how generous the regulator's budget is. If the budget is large so that a high level of quality

[1]See Ringard et al. (2016) for a detailed description of the reform.

can be implemented, welfare is higher in duopoly than in monopoly whenever entry pays off. More interestingly, the equilibrium outcome corresponds to the constrained welfare optimum whenever the private provider enters the market. If the budget is small, the market outcome is characterized by minimum quality levels. In this case, it might be optimal for the regulator not to use the full budget. The intuition is that the private provider locates at the corner of the market (to dampen costly quality competition) if the regulator provides too generous reimbursement. This raises total transportation costs and hence lowers welfare.

Cremer et al. (1991) is the first paper analyzing mixed oligopoly in the context of horizontal production differentiation. In their paper, firms first choose location and then price in a Hotelling model with quadratic transportation cost. Public firms behave as if minimizing transportation cost, while private firms maximize profit. As to the case of mixed duopoly, they find that equilibrium locations minimize transportation cost, while corner locations obtain for a purely private duopoly. Ma and Burgess (1993) consider a model of price competition and price regulation in a model of horizontal and vertical product differentiation, emphasizing how quality decisions affect price competition. In their model, locations are taken as given at the end-points of the market.

There is a more recent literature that considers the effects of competition in price-regulated (hospital) markets where firms are located along a Hotelling line. However, no paper we know about analyzes the case of an endogenous asymmetric location.

Brekke et al. (2006) model competition in symmetric location and quality between two profit-maximizing hospitals in a price-regulated market. They analyze the equilibrium outcomes in markets where the product price is exogenous. Using an extended version of the Hotelling model, they assume that firms choose their locations and the quality of the product they supply. The optimal price set by a welfarist regulator is derived. If the regulator can commit to a price prior to the choice of locations, the optimal (second-best) price causes overinvestment in quality and an insufficient degree of horizontal differentiation (compared with the first-best solution) if the transportation cost of consumers is sufficiently high. Under partial commitment, where the regulator is not able to commit prior to location choices, the optimal price induces first-best quality, but horizontal differentiation is inefficiently high.

Building on Brekke et al. (2006), Herr (2011) considers a mixed duopoly with fixed symmetric locations. She compares the mixed duopoly to the public and the private profit-maximizing duopoly. Mixed duopoly can be optimal if the public firm is more efficient and competition is intense. The reason is that the regulated price in mixed duopoly is between that of the public and the private duopoly. The regulated price induces under-provision (over-provision) of quality of the more (less) efficient hospital relative to the first-best level of quality. However, if the public hospital is sufficiently more efficient and competition is fierce, a mixed duopoly is the preferred market design. The regulated price in mixed duopoly discourages over-provision of quality of the less efficient private hospital and – together with the non-profit objective – encourages an increase in quality of the more efficient public hospital.

Brekke et al. (2012) consider a market where two providers are located at the endpoints of the market, prices might be regulated, and providers are altruistic towards consumers. Moreover, providers can exert effort to reduce the cost of production. They show that while a constraint on profit distribution always leads to higher production cost, the effects on quality are ambiguous.

Bardey et al. (2012) consider competition in symmetric location and quality of two profit-maximizers. They differ from Brekke et al. (2006) in that quality increases the variable cost incurred by providers, and that the regulator sets both the regulated price and a cost reimbursement rate. They show that if the pure prospective payment leads to underprovision of quality and overdifferentiation, a mixed reimbursement system will be dominant.

2 The model

We consider a three stage game with three active players: the regulator, a public hospital and a private hospital. At stage 1 the regulator determines the reimbursement given to the hospitals. At stage 2, the private hospital decides whether and where to locate, taking the public hospital's location in the middle of the market as given. Finally, at stage 3, one or both of the hospitals choose quality level, and patients decide where to receive one unit of medical care.

Let the utility of the patients be

$$u = s + q_i - t\left(x - x_i\right)^2,$$

where $s > t > 0$ is exogenous gross utility of treatment, $q_i \geq \underline{q}$ is the utility of the observable quality of treatment by hospital $i = 1, 2$, $\underline{q}$ is the minimum quality level each provided has to offer (normalized to zero), t is a transportation cost parameter, and $x \in [0, 1]$ is the address of the patient choosing treatment at hospital i with address x_i, where $x_1 = 1/2 \leq x_2$. With out loss of generality, we assume the private hospital is located on the right of the public hospital. Patients observe the hospitals' location and their quality levels. The patients decide which hospital to visit in order to maximize their utility. Because of $s > t > 0$ all patients seek treatments.

Hospital i's objective is to choose quality ($i = 1, 2$) and location ($i = 2$) to maximize:

$$u_i = \left(P - k - cq_i\right) d_i + \alpha_i q_i d_i - F_i,$$

subject to a non-negative profit constraint and a non-negative quality level,

$$\begin{aligned} \left(P - k - cq_i\right) d_i - F_i &\geq 0, \\ q_i &\geq 0, \end{aligned}$$

where $P > 0$ is the reimbursed price, $d_i \in [0, 1]$ is demand of hospital i, parameters $k \geq 0$ and $c \in (0, 1)$ denote the cost of providing quantity and quality, respectively, and F_i is the fixed cost of entry. Since the public hospital is already present in the market we assume that $F_2 > F_1 = 0$. The parameter, $\alpha_i \geq 0$ is the altruism parameter. It captures the fact that health care workers value the output they produce (i.e. the quality of care patients' receive) (Delfgaauw and Dur, 2002; and Glazer, 2004, Kaarboe and Siciliani, 2011, Siciliani et al, 2013). We assume that the public hospital cares more about the patients' utility than the private hospital, i.e. $\alpha_1 > \alpha_2$. For simplicity we set $\alpha_2 = 0$. Hence, the objective of the private hospital is to maximize profit, while the public hospital maximizes a weighted sum of profit and patient utility. Without loss of generality we set $k = 0$, and express the objective functions of the hospitals as

$$u_i = (P - (c - \alpha_i)\, q_i)\, d_i - F_i. \tag{1}$$

Notice that the altruistic component effectively lowers the marginal cost of providing quality for the public hospital. Obviously, if altruism is pronounced, $\alpha_1 \geq c$, then every unit of quality provided by the public hospital increases its utility. Consequently, every patient treated increases utility no matter which level of quality is provided. In this case quality is not a concern for the regulator (since the public hospital always chooses the highest quality level subject to the zero-profit constraint). Therefore we assume $\alpha_1 < c$. For the ease of exposition, we present the analysis for the case of $\alpha_1 < 3c/4$. Regarding the case $\alpha_1 \in [3c/4, c)$ qualitatively similar results obtain.

Obviously, non-negative profit, $\pi_i \geq 0$, puts an upper bound on quality, $q_i \leq P/c$. Consequently, we can rewrite hospital i's constraint maximization problem as:

$$\max_{q_i \in \left[0, \frac{P}{c}\right]} (P - (c - \alpha_i)\, q_i)\, d_i - F_i.$$

The demand of the two hospitals depends on locations and quality levels. If both hospitals choose *identical locations*, $x_1 = x_2$, then patients select the hospital with the highest quality. If both qualities coincide patients distribute evenly. Correspondingly, demand of hospital 1 and 2 is given by

$$d_1 = \begin{cases} 0 & \text{if } q_1 < q_2 \\ \frac{1}{2} & \text{if } q_1 = q_2 \\ 1 & \text{if } q_1 > q_2 \end{cases} \tag{2}$$

and $d_2 = 1 - d_1$, respectively.

On the other hand, if the private hospital locates away from the public hospital, $x_1 \neq x_2$, then the patient $\widehat{x}$ that is indifferent between seeking treatment at hospital 1 and hospital 2 is characterized by:

$$\begin{aligned} s + q_1 - t\,(\widehat{x} - x_1)^2 &= s + q_2 - t\,(\widehat{x} - x_2)^2 \\ \Longleftrightarrow \qquad \widehat{x} &= \frac{x_1 + x_2}{2} + \frac{q_1 - q_2}{2t\,(x_2 - x_1)} \\ \Longleftrightarrow \qquad \widehat{x} &= \frac{S}{2} + \frac{q_1 - q_2}{2t\Delta}, \end{aligned}$$

where $S \equiv x_1 + x_2$ and $\Delta \equiv x_2 - x_1$.

The demand of hospitals 1 and 2 is then

$$\begin{aligned} d_1 &= \max\{0, \min\{1, \widehat{x}\}\} \\ &= \min\{1, \max\{0, \widehat{x}\}\}. \end{aligned}$$

and $d_2 = 1 - d_1$, respectively. Clearly, demand of hospital $i = 1, 2$ increases in

own quality, q_i, and decreases in the quality level of the competitor.

The regulator cares about welfare in the health care sector. Welfare is defined as patients' utility net of transportation costs plus profit minus reimbursement. We assume that the regulator is given a fixed budget $B > 0$ by the central government and that quality is non-verifiable. Correspondingly, the regulator chooses the reimbursement price, $P \in [0, B]$, to maximize welfare,

$$\begin{aligned} W &= \int_0^{d_1} \left(q_1 - t(x_1 - x)^2\right) dx + \int_{d_1}^1 \left(q_2 - t(x_2 - x)^2\right) dx \\ &\quad + s + \sum_i (P - cq_i) d_i - P - F_2, \end{aligned} \tag{3}$$

subject to a constrained budget B, i.e. $P \leq B$.

3 Quality competition

Solving the model by backward induction, we start with deriving the Nash equilibrium of the quality subgame.

Let the price $P > 0$ and locations $x_1 = 1/2 \leq x_2$ be given, and consider first the case of center locations, i.e., $x_1 = x_2 = 1/2$. In this case the unique Nash equilibrium has both hospitals implementing

$$q_1^* = q_2^* = \frac{P}{c},$$

and earning zero (running) profit.[2] The utility of the public provider is $P\alpha_1/(2c)$.

Now consider the case where the private hospital does not locate at the center, i.e. $1/2 < x_2$. In this case we have

$$\begin{aligned} u_1 &= (P - (c - \alpha_1) q_1) d_1 \\ &= \frac{1}{2t\Delta} (P - (c - \alpha_1) q_1)(q_1 - q_2 + St\Delta). \end{aligned}$$

Maximizing u_i subject to the non-negative profit constraint for hospital 1 and 2 yields the following first order conditions for interior candidates:

$$\frac{\partial u_i}{\partial q_i} = \frac{\partial (P - (c - \alpha_i) q_i)}{\partial q_i} d_i + (P - (c - \alpha_i) q_i) \frac{\partial d_i}{\partial q_i} = 0,$$

[2]Notice that private hospital's fixed cost of enty is sunk at this point.

for $i, j = 1, 2$, and $i \neq j$. By solving $\partial u_i / \partial q_i = 0$ for q_i, we obtain the best reply functions of the two hospitals for interior candidates[3]:

$$\begin{aligned} q_1^{\mathrm{FOC}}(q_2) &= \frac{P}{2(c-\alpha)} - \frac{t\Delta S}{2} + \frac{1}{2}q_2 \\ q_2^{\mathrm{FOC}}(q_1) &= \frac{P}{2c} - \frac{t\Delta(2-S)}{2} + \frac{1}{2}q_1. \end{aligned} \tag{4}$$

Notice that the degree of altruism $\alpha \equiv \alpha_1$, the other parameters c and t, the location of the private hospital x_2, and the price P only affect the interceptions, but not the slope as long as both q_1 and q_2 assume interior values $q_i \in (0, P/c)$.

Taking the constraints of non-negative quality and profit into account, we get

$$q_i^{\mathrm{BR}}(q_j) = \max\left\{0, \min\left\{q_i^{\mathrm{FOC}}(q_j), \frac{P}{c}\right\}\right\}, \tag{5}$$

for $i, j = 1, 2$ and $j \neq i$.

Moreover, notice that, for non-center locations of the private hospital, $x_2 \in (1/2, 1]$, we have $\Delta > 0$ and $S \leq 3/2$. This implies $q_2^{\mathrm{FOC}}(P/c) < P/c$, i.e. the private hospital always earns positive profit. Therefore, hospital 2's best response (5) reduces to

$$q_2^{\mathrm{BR}}(q_1) = \max\left\{0, q_2^{\mathrm{FOC}}(q_1)\right\} \qquad \forall q_1 \in [0, P/c]. \tag{6}$$

Proposition 1 (Existence and uniqueness of a quality equilibrium) *For any $x_2 \in [\frac{1}{2}, 1]$, there exists a unique Nash equilibrium (q_1^*, q_2^*) of the quality subgame.*

Proof. See the Appendix.

Depending on the strength of altruism α, on the price P, and on the location of the private hospital x_2, different types of Nash equilibria occur in the different quality subgames $x_2 \in (1/2, 1]$. We classify these Nash equilibria as follows:

Definition 1 *Suppose (q_1^*, q_2^*) represents a Nash equilibrium of the quality subgame. We distinguish the following* equilibrium types*:*
Type I: *Both hospitals choose zero quality, i.e. $q_1^* = q_2^* = 0$.*
Type II: *One and only one hospital implements positive quality. If $q_2^* = 0 < q_1^* < P/c$, we refer to this case as a* type IIa *equilibrium, while $q_1^* = 0 < q_2^*$ is called a* type IIb *equilibrium.*[4]
Type III: *The quality equilibrium is interior, i.e. $q_i^* \in (0, P/c)$ for both hospitals $i = 1, 2$.*
Type IV: *The public hospital's equilibrium quality level is constrained by its non-negative profit condition, $q_1^* = P/c$, and $q_2^* = q_2^{BR}(P/c)$.*
Correspondingly, we say a location x_2 lies in equilibrium region $\tau \in \mathcal{T} \equiv \{I, IIa, IIb, III, IV\}$*, denoted by $x_2 \in X_\tau$, if location x_2 gives rise to a quality equilibrium of the corresponding type $\tau \in \mathcal{T}$.*

[3] The second order condition for a profit maximum is satisfied for all $x_2 > 1/2$ since $\partial^2 u_i / \partial q_i^2 < 0$.

[4] Recall that $q_2^{BR}(q_1) < P/c$ for all $q_1 \in [0, P/c]$. Hence, both in an equilibrium of type IIa and of type IIb equilibrium, the equilibrium quality is strictly below P/c for *both* providers.

A higher type number corresponds to a higher level of quality. Equilibrium types IIa and IIb differ with regard to which hospital implements positive quality. Figure 1 displays the different types of quality equilibrium. Parts (a) to (e) of Figure 1 correspond to equilibrium types I, IIa, IIb, III, and IV, respectively.

Figure 1 to be included here (see pp. 30-34)

As it turns out, whether type IIa or type IIb occurs crucially depends on which hospital has lower marginal cost of quality at symmetric quality profiles, i.e. for $q_1 = q_2$. This motivates the following definition:

Definition 2 *The private (public) hospital has* lower initial cost of raising quality at (location) x_2 *if and only if*

$$(c-\alpha)\left. d_1\right|_{q_1=q_2=0} \overset{(<)}{>} c \left. d_2\right|_{q_1=q_2=0}$$

Observe that the private (public) hospital has lower initial cost of raising quality at x_2 when altruism is sufficiently weak, $\alpha < (>)\, 2c\,(S-1)\,/S$, or, equivalently, when the private hospital locates sufficiently away from the center, i.e. for

$$x_2 \overset{(<)}{>} \frac{2+\alpha/c}{4-2\alpha/c} =: \widehat{x}. \tag{7}$$

The next proposition characterizes the boundaries of the different equilibrium types (for technical details see the Appendix). In particular, it shows that, for a given location x_2, equilibrium type IIa (IIb) can only occur when the public (private) hospital has lower initial cost of raising quality.

Proposition 2 (Equilibrium types by size of the price) *Depending on the price P, on the location of the private hospital, x_2, and on the public hospital's degree of altruism, α, all equilibrium types can occur. For a low price P, the equilibrium is of type I, and both hospitals provide zero quality. For a high price P, the equilibrium is of type IV, and the public hospital implements maximum quality P/c. For intermediate levels of P, the equilibrium will be of type IIa, IIb, or III, such as detailed below:*

(a) Suppose the public hospital has lower initial cost of raising quality, i.e. $x_2 < \widehat{x}$. Then, with increasing size of the price, the equilibrium is of type I, IIa, III, and IV.

(b) Suppose the private hospital has lower initial cost of raising quality, i.e. $x_2 > \widehat{x}$. Then, with increasing size of the price, the equilibrium will be of type I, IIb, III, and IV.

(c) If neither hospital has lower initial cost of raising quality, i.e. if $x_2 = \widehat{x}$, then, with increasing size of the price, the equilibrium will be of type I, III, and IV.

Proof. See the Appendix.

Some remarks are in order. Firstly, taking the location x_2 of the private hospital as given, a higher price P increases the intensity of quality competition and hence leads to higher equilibrium levels of quality. More specifically, for a low price no hospital provides positive quality and equilibrium type I occurs. For a slightly higher price equilibrium type II results. Whether it is of type IIa or of type IIb, depends on which hospital has lower initial cost of raising quality, $(c-\alpha)\, d_1 = (c-\alpha)\, S/2$ or $cd_2 = c(2-S)/2$, at $q_1 = q_2 = 0$. When altruism is weak (low α) or locations are distant ($x_2 > \widehat{x}$) then the private hospital has lower initial cost of raising quality and equilibrium type IIb is reached. On the other hand, for $x_2 < \widehat{x}$ the public hospital has lower initial cost of raising quality and equilibrium type IIa is realized.[5]

The inequality $x_2 > \widehat{x} = (2c+\alpha) / (4c-2\alpha)$ can only be satisfied for some $x_2 \in (1/2, 1]$ if α is not too large, viz. for $\alpha \leq 2c/3$. Put differently, for $\alpha > 2c/3$ it will always be equilibrium type IIa that is reached at the boundary to equilibrium type I, no matter where hospital 2 decides to locate. For $\alpha \leq 2c/3$ and $x_2 \leq (2c+\alpha) / (4c-2\alpha)$ type IIa is realized as well, while, for $\alpha \leq 2c/3$ and $x_2 > (2c+\alpha) / (4c-2\alpha)$, equilibrium type IIb occurs. In this latter case, a small increase in quality by the private hospital does not entail a quality reaction by the public hospital. This happens because competition is sufficiently weak as locations are distant and altruism is weak as well. For higher prices, the equilibrium is of type III or IV.

Secondly, in most cases it is the public hospital which delivers the highest quality level. Only when the private hospital has lower initial cost of raising quality and for some intermediate range of prices P such that the equilibrium is of type IIb or III, the private provider chooses higher quality in equilibrium. This case occurs when the public provider's altruism is weak or, equivalently, when the hospitals are located far from each other.

Finally, in all the cases (a)-(c), a high price will eventually lead to a type IV equilibrium no matter where the private hospital has chosen to locate. In the Appendix we show that the lowest price for this to happen is

$$P = \frac{7tc\,(c-\alpha)}{8\alpha}.$$

This motivates the following definition:

Definition 3 *The price is called*

- *(a)* high *when* $P/\,(ct\,(c-\alpha)) \geq 7/\,(8\alpha)$*;*
- *(b)* low *when* $P/\,(ct\,(c-\alpha)) < 5/\,(9\alpha)$*;*
- *(c)* intermediate *when* $P/\,(ct\,(c-\alpha)) \in [5/\,(9\alpha)\,, 7/\,(8\alpha))$.

[5] Notice that for the case $\alpha_1 \in [3c/4, c)$ none of the interior quality equilibrium types IIa or IIb occurs.

A high price implies an equilibrium of type IV no matter where the private hospital locates. A low price entails that no candidate for a local profit maximum lies in the interior of equilibrium region IV. For intermediate prices, there exist two candidate locations for the private provider: an interior location in region IV and a boundary location, which lies at the (i) boundary of regions I and IIa or (ii) at the boundary of regions IIb and III, or (iii) at the boundary of the location space, i.e. $x_2 = 1$.

The corollary below collects these properties and that the equilibrium type varies monotonically with the location x_2 of the private hospital. The results directly follow from the proof of Proposition 2.

Corollary 1 (Equilibrium types by location) *Let the price $P > 0$ be fixed and the location $\widehat{x}$ given by (7). Then the following statements hold true:*

(a) For any $x_2 < \widehat{x}$, a decrease in x_2 (weakly) increases the equilibrium type $\tau \in \{I, IIa, III, IV\}$. For $\Delta = x_2 - 1/2 > 0$ sufficiently small, equilibrium type IV is induced.

(b) For any $x_2 > \widehat{x}$, an increase in x_2 (weakly) decreases the equilibrium type $\tau \in \{I, IIb, III, IV\}$. Depending on the price and on parameter values, not all equilibrium regions may be reached.

(c) If the price is high then all locations belong to equilibrium region IV.

(d) If the price is intermediate, the location $x_2 = 5/6$ belongs to equilibrium region IV and gives rise to a local profit maximum of the private hospital. Moreover, the location $x_2 = 1$ does not belong to region IV.

(e) If the price is low and $\alpha/c > 2/3$ then $\widehat{x} > 1$ and part (a) applies to all $x_2 \in (1/2, 1]$. If the price is low and $\alpha/c < 2/3$ then $\widehat{x} < 1$. In this case, part (a) applies to all $x_2 \in (1/2, \widehat{x})$ and part (b) to all $x_2 \in (\widehat{x}, 1]$.

The above corollary proves helpful when determining the private hospital's locational choice.

4 The private hospital's location choice

We continue with analyzing stage 2. At this stage, the private hospital decides whether to enter and, if so, where to locate. When making this decision the private hospital takes the public hospital's location in the middle of the market as given and anticipates the consequences of its location choice for the quality equilibrium at stage 3. To derive the private hospital's location, we first compare locations within equilibrium types of the quality subgame. Subsequently, we compare across equilibrium types.

The private hospital chooses its location $x_2 \in [1/2, 1]$ in order to maximize profit

$$\pi_2 = (P - cq_2)\, d_2.$$

Hence, marginal profit is given by[6]

$$\begin{aligned}\frac{\partial \pi_2}{\partial x_2} &= \frac{\partial}{\partial x_2}\left(\left(P - cq_2\right) d_2\right) \\ &= -c\frac{\partial q_2}{\partial x_2} d_2 + \left(P - cq_2\right)\frac{\partial d_2}{\partial x_2},\end{aligned}$$

where q_2, d_2, and the derivatives depend on the equilibrium type under consideration.

Exemplarily, consider the locational choice within *equilibrium type I.* In this case, we have $q_1^* = q_2^* = 0$ and $d_2^* = 1 - S/2$. Hence, the private hospital can affect its demand only through a change in its location. As hospital 2's demand decreases in x_2, we have

$$\frac{\partial \pi_2}{\partial x_2} < 0.$$

Thus, within equilibrium type I, hospital 2 will locate as close to the center as possible.

The other cases are analyzed in the Appendix. Proposition 3 below summarizes our findings.

Proposition 3 (Locational choice within equilibrium types) *The private hospital's profit increases (decreases) with its location* x_2*,*

$$\frac{\partial \pi_2}{\partial x_2} > (<) 0,$$

for equilibrium types IIa and III (I and IIb). Within equilibrium type IV we have

$$\frac{\partial \pi_2}{\partial x_2} \gtreqless 0 \Leftrightarrow x_2 \lesseqgtr \frac{5}{6}.$$

Proof. See the Appendix.

Like in the standard Hotelling model, such as analyzed in d'Aspremont et al. (1979), our model exhibits a trade-off between moving away from the competitor to soften competition (the *competition effect*) and getting closer to steal demand (the *demand effect*).

Consider a marginal shift of the private hospital's location to the left, i.e. a small decrease in x_2. In an equilibrium of type I or IIb, the public hospital does not respond by changing its quality, that is, the competition effect is weak. Accordingly, the private hospital moves closer to the public hospital to steal demand from the public hospital. In contrast, in an equilibrium of type IIa or III the public hospital responds by raising quality when the private hospital gets closer. Since quality competition is costly, the private hospital locates further away from the center to dampen quality competition. Finally, in an equilibrium

[6] Even though, at stage 2, the (reduced) profit function represents a function of a single variable, we use the standard notation for partial derivatives to avoid misunderstandings, given that we have already used the symbol d to denote demand.

of type IV the public hospital's zero profit constraint is binding so that it chooses a fixed quality level that does not (locally) change with the private hospital's location. In this case the private hospital trades off increased demand and lower quality cost. It turns out that provider 2's optimal location within equilibrium type IV is 5/6.

We continue our analysis comparing locations across equilibrium types. Firstly, consider the case of a high price. In this case, the above proposition shows that the optimal location is $x_2^* = 5/6$, since all locations $x_2 \in (1/2, 1]$ give rise to an equilibrium of type IV (entailing positive profit, while profit would be zero at $x_2 = 1/2$).

Secondly, consider the case of a low price. By Corollary 1, it depends on the signs of $\alpha/c - 2/3$ and $x_2 - \widehat{x}$, which equilibrium types the private hospital can induce by choosing its location $x_2 \in (1/2, 1]$. If $\alpha/c \geq 2/3$ then the private hospital's optimal location either corresponds to the boundary of equilibrium regions I and IIa or, if this boundary lies outside the feasible range $[1/2, 1]$, the private hospital resides at the corner $x_2 = 1$. For, marginal payoff of the private hospital is increasing in x_2 within regions IV, III and IIa, and decreasing within region I.

On the other hand, if $\alpha/c < 2/3$ then the private hospital optimally locates at the boundary of regions I and IIa (for P suff. low). It resides at the boundary of regions IIb and III (for moderately low P), and it chooses the corner location $x_2 = 1$ if the price is not too low.

We summarize the above discussion in the following proposition. The exact conditions and the proof are deferred to the Appendix.

Proposition 4 (Locational choice across equilibrium types)
(a) If the price is high then the private hospital locates at $x_2 = 5/6$.
(b) Suppose the price is low and altruism strong ($\alpha/c \geq 2/3$)*. Then the private hospital locates at the boundary of regions I and IIa if altruism is sufficiently strong or the price sufficiently low. Otherwise, it locates at the corner* $x_2 = 1$.
(c) Suppose the price is low and altruism weak ($\alpha/c < 2/3$)*. Then the private hospital locates at the boundary of regions I and IIa if the price is sufficiently low; it locates at the boundary of regions IIb and III if the price is moderately low; and it locates at the corner* $x_2 = 1$ *if the price is not too low.*

Figure 2 displays three cases of locational choices that can occur. Parts (a) to (c) of Figure 2 correspond to cases (a) to (c) in Proposition 4.

Figure 2 to be included here (see p. 35)

When *the price is high*, the competition effect is weak. This follows because the zero profit constraint of the public provider is binding and because it implements maximum quality no matter where the private hospital locates. The competition effect is also weak when compared to that of a standard Hotelling model in the vein of d'Aspremont et al. (1979). In contrast, the demand effect *is comparable* to the standard Hotelling model. This explains why maximum differentiation no longer results in this case.[7] When *the price is low*, the public hospital would respond by raising quality if the private hospital got closer to the center. This reinforces the competition effect relative to the case of a high price so that a corner location can result.

For *intermediate prices* more candidate locations and many subcases need to be considered to rank profits of the private provider. However, also for intermediate prices the equilibrium location of the private hospital is generically unique. More specifically, with increasing price first the equilibrium location moves towards the right corner. Eventually, the location $x_2 = 5/6$ becomes a local profit maximum in addition to the other local profit maxima. Similar to the above proposition, these candidates will either be at the boundary of regions I and IIa, at the boundary of regions IIb and III, or at the corner $x_2 = 1$. When the price increases further, it eventually reaches a level where provider 2 is indifferent between location $x_2 = 5/6$ and one of the other candidates. For larger prices, provider 2 locates at $x_2 = 5/6$.

5 Welfare comparison between market forms

The regulator should open up the market if welfare in duopoly exceeds welfare in monopoly. Recall the welfare function (3) which is to be evaluated subject to a constrained budget B, i.e. $P \leq B$. We rewrite the welfare function as

$$W = \sum_{i=1,2} (1-c)\, q_i d_i - \int_0^{d_1} t \left(\frac{1}{2} - x\right)^2 dx - \int_{d_1}^{1} t\,(x_2 - x)^2\, dx + s - F_2. \quad (8)$$

Because of $\alpha < c$, the public hospital produces zero quality in the case of no entry no matter how large the budget is. This follows since quality is assumed to be non-verfiable. Accordingly, welfare in the case of monopoly amounts to[8]

$$W^m = s - t \int_0^1 \left(\frac{1}{2} - x\right)^2 dx = s - \frac{t}{12}.$$

We continue with the case of duopoly. Clearly, quality under duopoly is weakly higher than under monopoly. It is strictly higher whenever the budget allows to set the price sufficiently high so that the private provider's location x_2 gives rise to a quality equilibrium of type II, III, or IV.[9]

[7] Technically, the competition effect is linear in our model, whereas the demand-stealing effect is quadratic. In contrast, both effects are quadratic in the Hotelling model analyzed by d'Aspremont et al. (1979).

[8] Notice that the center location maximizes welfare (or minimizes transportation cost) in the case of monopoly.

[9] For quality equilibria of type IIa and IIb, quality is strictly higher for patients treated by the public and the private hospital, respectively. For equilibria of type III and IV quality is higher *for all* patients.

Consider the case of a *small budget,* which only allows to set a low price, i.e. $B < 5tc(c-\alpha)/(9\alpha)$. We show that it can be optimal for the regulator not to use the entire budget. We content ourselves with providing the intuition. A formal analysis is given in the Appendix.

When the budget only allows to set a low price, it follows from Proposition 4 that the private hospital locates at the boundary of regions I and IIa if the price is sufficiently low. In this case, both providers implement zero quality, $q_1^* = q_2^* = 0$, and the exact location of the private provider depends on the price P. The private provider locates close to the center if the price is low and resides further to the right if the price is high (but still below B). Since both providers implement zero quality, the first term in (8) is zero so that maximizing social welfare becomes equivalent to minimizing transportation cost. Notice that transportation cost decreases for $x_2 < 5/6$ and increases for $x_2 > 5/6$.[10] It hence follows that the regulator (a) spends the full budget, $P^* = B$, as long as $x_2(P^*) < 5/6$ and (b) optimally implements location $x_2 = 5/6$ whenever the budget allows to to set the price accordingly. Observe that, in order to implement $x_2 = 5/6$, the regulator has to set the price $P^* = 4t(c-\alpha)/9$. Thus, whenever the budget is small but exceeds P^* the regulator optimally withholds the amount $B - P^*$.

We continue to compare welfare in monopoly and duopoly. It is obvious that, for a given level of entry cost F_2, opening up the market only pays off if the reduction in transportation cost exceeds F_2. If the budget is too small (as in case (a) above), the reduction in transportation cost is small as well so that opening up the market does not pay off to the society. In case (b), the location $x_2^* = 5/6$ results in a market share of $d_2^* = 1/3$ of the private provider so that the transportation cost in duopoly reduces to

$$TC^d = \int_0^{2/3} t\left(\frac{1}{2} - x\right)^2 dx + \int_{2/3}^1 t\left(\frac{5}{6} - x\right)^2 dx = \frac{5t}{108}.$$

Comparing with the transportation cost in monopoly, we obtain

$$\Delta TC \equiv TC^m - TC^d = \frac{t}{12} - \frac{5t}{108} = \frac{t}{18} > 0.$$

Hence, opening up the market entails higher welfare if $F_2 < t/18$. Notice that entry only occurs if the private provider earns positive profit, i.e. $\pi_2^* = P^* d_2^* - F_2 \geq 0$. In case (b), this inequality reduces to $F_2 \leq 4t(c-\alpha)/27$.

The following proposition summarizes our findings for the case of a small budget. The exact conditions and the proof are provided in the Appendix.

Proposition 5 (Welfare comparison for small budgets) *Suppose the budget B is small and altruism sufficiently strong.*

[10] In the standard Hotelling model total transportation cost is minimized by splitting the Hotelling street into two parts and then putting firms at the center of each segment, i.e. for $x_1 = 1/4$ and $x_2 = 3/4$. In our case, where $x_1 = 1/2$ is given, the transportation cost is minimized by splitting the right half of the Hotelling street into three parts of identical length $1/6$ and then putting provider 2 at $x_2 = 5/6$.

(a) If $B \leq 4t(c-\alpha)/9$ then the regulator spends the full budget, setting $P^* = B$. If the entry cost F_2 exceeds $t/18$, then welfare is higher in monopoly than in duopoly.
(b) For $B \in (4t(c-\alpha)/9, 5tc(c-\alpha)/(9\alpha))$, the regulator does not spend the full budget, setting $P^* = 4t(c-\alpha)/9$. In this case, welfare is (weakly) higher in duopoly than in monopoly if $F_2 < \min\{t/18, 4t(c-\alpha)/27\}$.
In both cases, equilibrium quality is at the minimum level and the provider locates the further to the right the higher the regulator sets the price.

Consider the case of a *large budget* that allows to set a high price, i.e. $B \geq 7tc(c-\alpha)/(8\alpha)$. For a high price $P \in [7tc(c-\alpha)/(8\alpha), B]$, we have $x_2^* = 5/6$, $q_1^* = P/c$, $q_2^* = P/c - t/9$, $d_1^* = 5/6$, $d_2^* = 1/6$, $\pi_1^* = 0$, and $\pi_2^* = ct/54$. Accordingly, the private provider enters whenever $F_2 \leq ct/54$. Moreover, both quality levels increase in the price P, while the quality difference and equilibrium demand do not depend on it. Interestingly, the equilibrium profit of neither firm depends on P. Hence, any increase in the price entails an increase in patient benefit and social welfare. To see this, we insert the above equilibrium expressions into (3) to obtain the social welfare in the case of duopoly:

$$\begin{aligned} W^d &= (1-c)\left(\frac{P}{c} - \frac{t}{54}\right) - t\int_0^{\frac{5}{6}} \left(\frac{1}{2} - x\right)^2 dx - t\int_{\frac{5}{6}}^1 \left(\frac{5}{6} - x\right)^2 dx + s - F_2 \\ &= (1-c)\left(\frac{P}{c} - \frac{t}{54}\right) - \frac{1}{18}t + s - F_2. \end{aligned} \tag{9}$$

Therefore, social welfare increases in the size of the budget when the entire budget is spent, i.e. for $P = B$.

In order to compare welfare in the case of duopoly with that in the case of monopoly, notice that the lowest budget compatible with a high price is $B = 7tc(c-\alpha)/(8\alpha)$. By inserting the price $P = 7tc(c-\alpha)/(8\alpha)$ and the highest possible entry cost compatible with duopoly, $F_2 = ct/54$, we get

$$\begin{aligned} W^d - W^m &\geq (1-c)\left(\frac{7t(c-\alpha)}{8\alpha} - \frac{t}{54}\right) - \frac{ct}{54} + \frac{t}{36} \\ &= \frac{t}{216}\left(189\frac{c}{\alpha}(1-c) + (189c - 187)\right) \\ &> \frac{t}{108}, \end{aligned}$$

where in the last row we have used $\alpha < c$. Thus, whenever the private hospital finds it optimal to enter the market, entry increases social welfare. There is no excess entry. The next proposition summarizes our findings for the case of a large budget.

Proposition 6 (Welfare comparison for large budgets) *Suppose the budget is large and the price is high, i.e. $B \geq 7tc(c-\alpha)/(8\alpha)$ and $P \in [7tc(c-\alpha)/(8\alpha), B]$. Then we have:*

(a) Welfare is higher in duopoly than in monopoly whenever entry pays off.
(b) When the private provider enters the market, the equilibrium outcome corresponds to the constrained welfare maximum.

Let us compare the market solution with the *constrained welfare maximum*, i.e. the maximum welfare that is obtainable when quality is non-verifiable and the regulator can write a contract on the private provider's location. If the budget is small, the constrained welfare maximum corresponds to the market solution characterized in Proposition 5(b). This follows because both providers implement minimum quality levels, $q_1 = q_2 = 0$ so that the constrained welfare maximum obtains when the transportation cost is minimized.

If the budget is large, we find that the constrained welfare optimum corresponds to the market solution as well, that is, the regulator would like the private provider to locate at $x_2 = 5/6$. This follows since the quality difference and the private provider's profit are independent of the private provider's location. Hence, the welfare trade-off boils down to the loss in patient utility due to lower quality of the private provider and saved transportation cost. Since the difference in quality is $t/9$ and since it is independent of P, the private provider chooses a location such that the corresponding marginal consumer's transportation cost is equal to $t/9$. As it turns out, this implies $x_2 = 5/6$, because transportation cost is quadratic and since the corresponding marginal consumer is located at $x = 5/6$. For, it is this consumer who travels a distance of 1/3, incurring a transportation cost of $t\,(5/6 - 1/2)^2 = t/9$.

Finally, for intermediate prices, welfare is higher in duopoly than in monopoly if the entry cost is not too high. This follows since the private provider always locates to the right of the center so that the transportation cost is higher in monopoly. Furthermore, equilibrium quality is weakly higher in duopoly, entailing an increase in patient utility.

6 Conclusion

In this paper we analyze the effects on quality and location of opening up the health care market for private for-profit providers. We find that opening up the market in most cases will increase the quality of the incumbent public provider. However, there are also cases where the private for-profit provider manages to dampen quality competition up to a point where quality remains unchanged. He does so by strategically locating away from the public provider. In some cases even maximum differentiation obtains, where the private provider locates at the endpoint of the market. These cases occur when the regulator offers low reimbursement prices.

On the other hand, if reimbursement is high, competition becomes more intense resulting in higher quality after entry. The private provider locates such that the public provider implements the highest quality level consistent with non-negative profit. In this case, the private entrant chooses lower quality, obtains a market share of 1/6, and earns positive profit.

The comparison of social welfare in monopoly and duopoly crucially depends on the regulator's budget. If the budget is low, quality levels do not respond to a change in the price. Hence, it might not be optimal to use the full budget, since a generous reimbursement generates a strategic response where the private provider moves to far towards the corner. If the budget is large and the private provider enters, the equilibrium outcome corresponds to the constrained welfare optimum. Moreover, welfare is higher than in monopoly, that is, there is no excess entry.

7 References

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8 Appendix

We now provide the proofs of our results.

Proof of Proposition 1. Consider $x_2 = 1/2 = x_1$ first. In this case, by discontinuity of the hospital-specific demand function (2), a standard Bertrand type of argument shows existence and uniqueness of the Nash equilibrium $\left(q_1^{NE}, q_2^{NE}\right) = (P/c, P/c)$.

Now consider $x_2 \in (1/2, 1]$. Notice first that the best reply curves (5) are weakly increasing. Therefore, Tarski's fixed-point theorem yields existence of a Nash equilibrium $\left(q_1^{NE}, q_2^{NE}\right)$ (Tarski, 1955). To show that this equilibrium is unique, the following Lemma turns out helpful.

Lemma 1 *Let $q_i^{BR}(q_j)$ be given by (5) and $q', q'' \in Q \equiv [0, \infty)^2$ such that $q' = (q_1', q_2')$ and $q'' = (q_1'', q_2'')$. Then we have:*
(a) If $q_1' = q_1^{BR}(q_2')$ and $q_2' \leq q_2''$ then $q_1^{BR}(q_2'') \leq q_1' + (q_2'' - q_2')/2$.
(b) If $q_2' = q_2^{BR}(q_1')$ and $q_1' \leq q_1''$ then $q_2^{BR}(q_1'') \leq q_2' + (q_1'' - q_1')/2$.

Proof of Lemma 1.. Choose $q', q'' \in Q$ arbitrarily.

To prove part (a), let $q_1' = q_1^{BR}(q_2')$ and $q_2' \leq q_2''$. We need to show that

$$q_1^{BR}(q_2'') \leq q_1^{BR}(q_2') + \frac{1}{2}(q_2'' - q_2'). \tag{10}$$

First, consider the cases $q_1^{FOC}(q_2'') \leq 0$ and $q_1^{FOC}(q_2') \geq P/c$. If $q_1^{FOC}(q_2'') \leq 0$, then $q_2' \leq q_2''$ implies that $q_1^{FOC}(q_2') \leq q_1^{FOC}(q_2'') \leq 0$ and hence $q_1^{BR}(q_2'') = q_1^{BR}(q_2') = 0$. Therefore, the inequality is satisfied. Similarly, if $q_1^{FOC}(q_2') \geq P/c$ then $q_2' \leq q_2''$ implies $q_1^{FOC}(q_2'') \geq q_1^{FOC}(q_2') \geq P/c$ and hence $q_1^{BR}(q_2'') = q_1^{BR}(q_2') = P/c$ so that the inequality is satisfied as well.

Second, consider $q_1^{FOC}(q_2'') \in (0, P/c)$ and observe that, by monotonicity of $q_1^{FOC}(\cdot)$, this implies $q_1^{FOC}(q_2') < P/c$. On the one hand, if $q_1^{FOC}(q_2') \in (0, P/c)$ then $q_1^{BR}(q_2'') - q_1^{BR}(q_2') = q_1^{FOC}(q_2'') - q_1^{FOC}(q_2') = \frac{1}{2}(q_2'' - q_2')$ and inequality (10) holds with equality. On the other hand, if $q_1^{FOC}(q_2') \leq 0$ then we have $q_1^{BR}(q_2'') = q_1^{FOC}(q_2'')$ and $q_1^{BR}(q_2') = 0$. In this case, inequality (10) reduces to

$$q_1^{FOC}(q_2'') = \frac{P}{2(c-\alpha)} - \frac{tS\Delta}{2} + \frac{1}{2}q_2'' \leq \frac{1}{2}(q_2'' - q_2'),$$

which is equivalent to $q_1^{FOC}(q_2') \leq 0$ and hence satisfied.

Third, consider the remaining case, i.e. $q_1^{FOC}(q_2'') \geq P/c$ and $q_1^{FOC}(q_2') < P/c$. On the one hand, if $q_1^{FOC}(q_2') \leq 0$ then notice that $q_1^{FOC}(q_2'') \geq P/c$ is equivalent to

$$\frac{P}{2(c-\alpha)} - \frac{tS\Delta}{2} + \frac{1}{2}q_2'' \geq \frac{P}{c}$$

and $q_1^{FOC}(q_2') \leq 0$ to

$$\frac{P}{2(c-\alpha)} - \frac{tS\Delta}{2} + \frac{1}{2}q_2' \leq 0.$$

Combining the two inequalities, we obtain

$$\frac{1}{2}\left(q_2''-q_2'\right)\geq\left(\frac{P}{c}-\frac{P}{2\left(c-\alpha\right)}+\frac{tS\Delta}{2}\right)+\left(\frac{P}{2\left(c-\alpha\right)}-\frac{tS\Delta}{2}\right)=\frac{P}{c},$$

which is equivalent to (10) given the case under consideration. On the other hand, if $q_1^{FOC}\left(q_2'\right)\in(0,P/c)$ then inequality (10) reduces to

$$\frac{P}{c}\leq\frac{P}{2\left(c-\alpha\right)}-\frac{tS\Delta}{2}+\frac{1}{2}q_2'',$$

which is equivalent to $q_1^{FOC}\left(q_2''\right)\geq P/c$ and hence satisfied.

Part (b) can be shown similarly. ■

We continue with proving uniqueness. Let $q',q''\in Q$ both be Nash equilibria of the quality subgame and suppose that $q_1'\leq q_1''$ (w.l.o.g.). On the one hand, as $q_2^{BR}\left(q_1\right)$ is weakly increasing, it follows that $q_2'\leq q_2''$. On the other hand, the above Lemma implies that

$$\begin{aligned} q_1'' &\leq q_1'+\frac{1}{2}\left(q_2''-q_2'\right) \qquad \text{and} \\ q_2'' &\leq q_2'+\frac{1}{2}\left(q_1''-q_1'\right). \end{aligned}$$

Rewriting the first inequality as

$$\frac{1}{2}\left(q_1''-q_1'\right)\leq\frac{1}{4}\left(q_2''-q_2'\right)$$

and combining it with the second inequality, we obtain

$$q_2''\leq q_2'+\frac{1}{4}\left(q_2''-q_2'\right),$$

which is equivalent to $q_2''\leq q_2'$. We have thus shown that $q_2'=q_2''$. Since the best reply correspondences are single-valued, it moreover follows that $q_1'=q_1^{BR}\left(q_2'\right)=q_1''=q_1^{BR}\left(q_2''\right)$ and hence $q'=q''$, which shows uniqueness of the Nash equilibrium. ■

Proof of Proposition 2. Let $P>0$ be arbitrary and let (q_1^*,q_2^*) be the unique Nash equilibrium of the quality game. Since the case $x_2=1/2(=x_1)$ was dealt with in Proposition 1, we restrict attention to $x_2\in(1/2,1]$. We show that, depending on P and $S=1/2+x_2$, different types of quality equilibria result. The following table provides an overview of this relationship:

Type	**Boundary conditions**	**Quality equilibrium**
Type I	$\frac{P}{tc(c-\alpha)} \leq \Delta \min\left\{\frac{S}{c}, \frac{2-S}{c-\alpha}\right\}$	$q_1^* = q_2^* = 0$
Type IIa	$\Delta\frac{S}{c} < \frac{P}{tc(c-\alpha)} \leq \Delta\frac{4-S}{3c-2\alpha}$	$q_2^* = 0 < q_1^* < \frac{P}{c}$
Type IIb	$\Delta\frac{2-S}{c-\alpha} < \frac{P}{tc(c-\alpha)} \leq \Delta\frac{S+2}{3c-\alpha}$	$q_2^* > q_1^* = 0$
Type III	$\Delta\max\left\{\frac{S+2}{3c-\alpha}, \frac{4-S}{3c-2\alpha}\right\} < \frac{P}{tc(c-\alpha)}$ and $\frac{P}{tc(c-\alpha)} < \Delta\frac{S+2}{2\alpha}$	$0 < q_1^* < \frac{P}{c}, q_2^* > 0$
Type IV	$\Delta\frac{S+2}{2\alpha} \leq \frac{P}{tc(c-\alpha)}$	$q_1^* = \frac{P}{c}$, $q_2^* = \frac{P}{c} - \frac{t\Delta(2-S)}{2}$

The proof is divided into two parts. We start with establishing the order of the boundaries of the different equilibrium types. Subsequently, we show that a set of boundary conditions of a certain type implies a quality equilibrium of the corresponding type.

Order of boundaries. Notice that $\alpha < c$ implies the interval of equilibrium type IIa is non-empty (empty), $S/c < (>) \left(4-S\right)/\left(3c-2\alpha\right)$, if and only if the interval of equilibrium type IIb is empty (non-empty), $\left(2-S\right)/\left(c-\alpha\right) > (<) \left(S+2\right)/\left(3c-\alpha\right)$, both of which is equivalent to $\alpha > (<) \, 2c\left(S-1\right)/S$, i.e. the public hospital has lower (higher) initial cost of raising quality In the degenerate case of $\alpha = 2c\left(S-1\right)/S$, neither an equilibrium of type IIa nor one of type IIb exists.

First, consider the case $\alpha > 2c\left(S-1\right)/S$, i.e. the boundary conditions of type IIb are empty and, for type IIa, we have

$$\frac{S}{c} < \frac{4-S}{3c-2\alpha} < \frac{S+2}{2\alpha}$$

where the second inequality follows from $\alpha/c < 3/4 < \left(S+2\right)/4$, i.e., with increasing price P, the boundary conditions of types I, IIa, III, and IV are passed through successively.

Subsequently consider the case $\alpha < 2c\left(S-1\right)/S$, i.e.

$$\frac{2-S}{c-\alpha} < \frac{S+2}{3c-\alpha} < \frac{S+2}{2\alpha}$$

where the second inequality follows from $\alpha < c$. Hence, with increasing P, the boundary conditions of types I, IIb, III, and IV are passed through.

Finally, consider the case $\alpha = 2c\left(S-1\right)/S$. In this case, we have

$$\frac{S}{c} = \frac{4-S}{3c-2\alpha} = \frac{2-S}{c-\alpha} = \frac{S+2}{3c-\alpha} < \frac{S+2}{2\alpha}.$$

By the above equalities, the equilibrium regions IIa and IIb are both empty. It hence follows that, with increasing P, the equilibrium is of type I, III, and IV, respectively.

Relationship of boundaries and equilibrium types. Since the quality equilibrium is unique by Proposition 1, it suffices to show that the boundary conditions of a certain type imply a quality equilibrium of the corresponding type.

Type I: Let P be such that

$$\frac{P}{tc\left(c-\alpha\right)} \leq \Delta \min\left\{\frac{S}{c}, \frac{2-S}{c-\alpha}\right\}.$$

This implies that the interceptions of both $q_1^{FOC}\left(q_2\right)$ and $q_2^{FOC}\left(q_1\right)$ in (4) are non-positive. It hence follows that $q_1^{BR}\left(0\right) = q_2^{BR}\left(0\right) = 0$, i.e. $\left(q_1^*, q_2^*\right) = \left(0, 0\right)$ represents an equilibrium of type I.

Type IIa: Let P satisfy

$$\Delta\frac{S}{c} < \frac{P}{tc\left(c-\alpha\right)} \leq \Delta\frac{4-S}{3c-2\alpha} \tag{11}$$

The two inequalities above imply $q_1^{FOC}\left(0\right) > 0$ and $q_2^{FOC}\left(q_1^{FOC}\left(0\right)\right) \leq 0$, respectively, i.e. we have $q_2^{BR}\left(q_1^{FOC}\left(0\right)\right) = 0$ and $q_1^{BR}\left(0\right) = \min\left\{q_1^{FOC}\left(0\right), P/c\right\}.$ Moreover, it follows from $\alpha/c < 3/4$ that $q_1^{FOC}\left(0\right) < P/c$, i.e. $q_1^{BR}\left(0\right) = q_1^{FOC}\left(0\right).$ To see this, notice that

$$\begin{aligned} q_1^{FOC}\left(0\right) \quad &= \quad \frac{P}{2\left(c-\alpha\right)} - \frac{t\Delta S}{2} < \frac{P}{c} \\ \iff \quad & \quad \frac{P}{tc\left(c-\alpha\right)\Delta}\left(2\alpha - c\right) < S \end{aligned} \tag{12}$$

For $\alpha \leq c/2$ this is clearly satisfied. Hence, consider $\alpha \in \left(c/2, 3c/4\right).$ In this case, the above inequality is equivalent to

$$\frac{P}{tc\left(c-\alpha\right)\Delta} < \frac{S}{2\alpha - c}.$$

Notice that $\alpha < 3c/4$ implies

$$\frac{4-S}{3c-2\alpha} < \frac{S}{2\alpha-c}. \tag{13}$$

Consequently, combining the inequalities (11) and (13), we obtain (12). Thus, $\left(q_1^*, q_2^*\right) = \left(q_1^{FOC}\left(0\right), 0\right)$ represents an equilibrium of type IIa.

Type IIb: Let P be such that

$$\Delta\frac{2-S}{c-\alpha} < \frac{P}{tc\left(c-\alpha\right)} \leq \Delta\frac{S+2}{3c-\alpha}.$$

Similarly to case IIa, we have $q_2^{FOC}\left(0\right) > 0$ and $q_1^{FOC}\left(q_2^{FOC}\left(0\right)\right) \leq 0$, which implies that $\left(q_1^*, q_2^*\right) = \left(0, q_2^{FOC}\left(0\right)\right)$ represents an equilibrium of type IIb.

Type III: Let P satisfy

$$\Delta \max\left\{\frac{S+2}{3c-\alpha}, \frac{4-S}{3c-2\alpha}\right\} < \frac{P}{tc\left(c-\alpha\right)} < \Delta\frac{S+2}{2\alpha}. \tag{14}$$

Consider a solution to the first order conditions (4),

$$\begin{aligned} q_1^{FOC} &= \frac{P(3c-\alpha)}{3c(c-\alpha)} - \frac{t\Delta(S+2)}{3} \\ q_2^{FOC} &= \frac{P(3c-2\alpha)}{3c(c-\alpha)} - \frac{t\Delta(4-S)}{3}. \end{aligned} \tag{15}$$

The left inequality in (14) implies that both quality levels are positive. Moreover, the right inequality in (14) is equivalent to $q_1^{FOC} < P/c$. Hence, $\left(q_1^{FOC}, q_2^{FOC}\right)$ represents an equilibrium of type III. By uniqueness of the quality equilibrium, we thus have $(q_1^*, q_2^*) = \left(q_1^{FOC}, q_2^{FOC}\right)$.

Type IV: If $P/(tc(c-\alpha)\Delta) \geq (S+2)/(2\alpha)$ then the solution to (4), given by (15), satisfies $q_1^{FOC} \geq P/c$ and hence $q_1^* = \min\left\{q_1^{FOC}, P/c\right\} = P/c$. Moreover, inserting $q_1^* = P/c$ into $q_2^{FOC}(q_1)$ we obtain $q_2^* = P/c - t\Delta(2-S)/2 > 0$. To see the inequality, notice first that the inequality is equivalent to

$$\frac{P}{tc(c-\alpha)\Delta} > \frac{2-S}{2(c-\alpha)}.$$

Secondly, observe that $a/c < 3/4$ implies $(S+2)c > 4\alpha$, which is equivalent to

$$\frac{S+2}{2\alpha} > \frac{2-S}{2(c-\alpha)}.$$

It hence follows that $P/(tc(c-\alpha)\Delta) \geq (S+2)/(2\alpha) > (2-S)/(2(c-\alpha))$, i.e. $q_2^* > 0$. We have thus shown that $(q_1^*, q_2^*) = (P/c, P/c - t\Delta(2-S)/2)$ represents an equilibrium of type IV. ■

Proof of Proposition 3. Let $P > 0$ and $x_2 \in (1/2, 1)$ be given arbitrarily and let $q^* = (q_1^*(x_2), q_2^*(x_2))$ denote the corresponding quality equilibrium that results at stage 3. Correspondingly, let $\pi_2(x_2) \equiv \pi_2(q_1^*(x_2), q_2^*(x_2))$ denote the reduced profit function of the private hospital at stage 2. The private hospital chooses x_2 in order to maximize profit, $\pi_2(x_2) = (P - cq_2^*(x_2))\, d_2^*(x_2)$, where equilibrium demand $d_2^*(x_2)$ is given by

$$d_2^*(x_2) = 1 - \frac{1/2 + x_2}{2} - \frac{q_1^*(x_2) - q_2^*(x_2)}{2t(x_2 - 1/2)}. \tag{16}$$

Hence, marginal profit $\pi_2'(x_2)$ is given by

$$\begin{aligned} \pi_2'(x_2) &= \frac{\partial}{\partial x_2}\left((P - cq_2^*)\, d_2^*\right) \\ &= -c\frac{\partial q_2^*}{\partial x_2} d_2^* + (P - cq_2^*)\frac{\partial d_2^*}{\partial x_2}. \end{aligned}$$

In the following, we determine the marginal profit of the private hospital, $\pi_2'(x_2)$, for each of the different equilibrium types separately. For any $\tau \in \mathcal{T} \equiv \{I, IIa, IIb, III, IV\}$, let X_τ and Q_τ denote the set of locations x_2 and the set of quality equilibria $q^*(x_2) = (q_1^*(x_2), q_2^*(x_2))$ corresponding to equilibrium type $\tau \in \mathcal{T}$, respectively.

Type I: Let $q^*(x_2) \in Q_I$ and let x_2 be an interior point in X_I, denoted as $x_2 \in int\ X_I$. Then, $q_1^*(x_2) = q_2^*(x_2) = 0$ implies $\partial q_2^*(x_2)/\partial x_2 = 0$, $d_2^*(x_2) = 3/4 - x_2/2$, and hence

$$\pi_2'(x_2) = P\frac{\partial d_2^*}{\partial x_2} < 0.$$

Type IIa: Let $q^*(x_2) \in Q_{IIa}$ and let $x_2 \in int\ X_{IIa}$, i.e. we have $q_2^*(x_1) = 0$ and $q_1^*(x_2) = q_1^{BR}(0) = P/(2(c-\alpha)) - t\Delta S/2 > 0$. Inserting $q_1^*(x_2)$ and $q_2^*(x_2)$ in (16), we get

$$\begin{aligned} d_2^*(x_2) &= 1 - \frac{S}{2} - \frac{q_1^*}{2t\Delta} \\ &= 1 - \frac{S}{4} - \frac{P}{4t\Delta(c-\alpha)} \end{aligned}$$

and hence

$$\begin{aligned} \frac{\partial d_2}{\partial x_2} &= \frac{\partial d_2}{\partial S}\frac{\partial S}{\partial x_2} + \frac{\partial d_2}{\partial \Delta}\frac{\partial \Delta}{\partial x_2} \\ &= -\frac{1}{4} + \frac{P}{4t\Delta^2(c-\alpha)}. \end{aligned}$$

This derivative is positive since $2q_1^*(x_2) = P/(c-\alpha) - t\Delta S > 0$ and $S \geq \Delta$ imply $P > t\Delta^2(c-\alpha)$. We thus obtain

$$\frac{\partial \pi_2}{\partial x_2} = P\frac{\partial d_2}{\partial x_2} > 0.$$

Type IIb: Let $q^*(x_2) \in Q_{IIb}$ and let $x_2 \in int\ X_{IIb}$, i.e. we have $q_1^*(x_2) = 0$ and $q_2^*(x_2) = q_2^{BR}(0) = P/(2c) - t\Delta(2-S)/2 > 0$. Inserting $q_1^*(x_2)$ and $q_2^*(x_2)$ in (16), we get

$$\begin{aligned} d_2^*(x_2) &= 1 - \frac{S}{2} + \frac{q_2^*}{2t\Delta} \\ &= \frac{P + ct\Delta(2-S)}{4ct\Delta}. \end{aligned}$$

Then profit $\pi_2(x_2)$ reduces to

$$\begin{aligned} \pi_2(x_2) &= \left(P - c\left(\frac{P}{2c} - \frac{t\Delta(2-S)}{2}\right)\right)\frac{P + ct\Delta(2-S)}{4ct\Delta} \\ &= \frac{(P + ct\Delta(2-S))^2}{8ct\Delta}, \end{aligned}$$

where we have used $S = 1/2 + x_2$ and $\Delta = x_2 - 1/2$. Calculating $\pi_2'(x_2)$, we obtain

$$\begin{aligned}\frac{\partial \pi_2}{\partial x_2} &= \frac{\partial \pi_2}{\partial S}\underbrace{\frac{\partial S}{\partial x_2}}_{=1} + \frac{\partial \pi_2}{\partial \Delta}\underbrace{\frac{\partial \Delta}{\partial x_2}}_{=1} \\ &= \frac{\partial \pi_2}{\partial S} + \frac{\partial \pi_2}{\partial \Delta} < 0,\end{aligned}$$

where the inequality follows from

$$\begin{aligned}\frac{\partial \pi_2}{\partial \Delta} &= -\frac{P^2 - t^2\Delta^2 c^2 (2-S)^2}{8t\Delta^2 c} < 0 \\ \text{and} \quad \frac{\partial \pi_2}{\partial S} &= -\frac{1}{4}\left(P + (2-S)\, t\Delta c\right) < 0.\end{aligned}$$

The numerator in the first row is positive because of $2cq_2^*(x_2) = P - ct\Delta (2-S) > 0$. Thus, $\pi_2'(x_2) < 0$.

Type III: Let $q^*(x_2) \in Q_{III}$ and let $x_2 \in int\ X_{III}$, i.e. quality equilibrium levels are given by (15):

$$\begin{aligned}q_1^*(x_2) &= \frac{P(3c-\alpha)}{3c(c-\alpha)} - \frac{t\Delta(S+2)}{3} \\ q_2^*(x_2) &= \frac{P(3c-2\alpha)}{3c(c-\alpha)} - \frac{t\Delta(4-S)}{3},\end{aligned}$$

which yields a quality gap of

$$q_1^*(x_2) - q_2^*(x_2) = \frac{\alpha P}{3c(c-\alpha)} - \frac{2t\Delta(S-1)}{3}.$$

Inserting the quality gap in (16), we get

$$d_2^*(x_2) = 1 - \frac{S}{2} - \frac{q_1^* - q_2^*}{2t\Delta} = \frac{2}{3} - \frac{S}{6} - \frac{P\alpha}{6ct\Delta(c-\alpha)}$$

and hence

$$\begin{aligned}\pi_2(x_2) &= (P - cq_2^*(x_2))\, d_2^*(x_2) \\ &= \left(P - c\left(\frac{P(3c-2\alpha)}{3c(c-\alpha)} - \frac{t\Delta(4-S)}{3}\right)\right)\left(\frac{2}{3} - \frac{S}{6} - \frac{P\alpha}{6ct\Delta(c-\alpha)}\right) \\ &= \frac{ct\Delta}{18}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)^2,\end{aligned}$$

where we have used that

$$\left(P - c\left(\frac{P(3c-2\alpha)}{3c(c-\alpha)} - \frac{t\Delta(4-S)}{3}\right)\right) = -\frac{P\alpha}{3(c-\alpha)} + \frac{ct\Delta(4-S)}{3}.$$

To establish $\pi_2'(x_2) > 0$, we exploit

$$
\begin{aligned}
\frac{\partial \pi_2}{\partial x_2} &= \frac{\partial \pi_2}{\partial S}\frac{\partial S}{\partial x_2} + \frac{\partial \pi_2}{\partial \Delta}\frac{\partial \Delta}{\partial x_2} = \frac{\partial \pi_2}{\partial S} + \frac{\partial \pi_2}{\partial \Delta} \\
&= -\frac{ct\Delta}{9}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right) \\
&\quad + \frac{ct}{18}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)\left((4-S) + \frac{\alpha P}{tc(c-\alpha)\Delta}\right) \\
&= \frac{ct}{18}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)\left(4 - 2\Delta - S + \frac{\alpha P}{tc(c-\alpha)\Delta}\right) \quad (17)
\end{aligned}
$$

where we have inserted

$$
\frac{\partial \pi_2}{\partial S} = -\frac{ct\Delta}{9}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)
$$

and

$$
\begin{aligned}
\frac{\partial \pi_2}{\partial \Delta} &= \frac{ct}{18}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)^2 + \frac{ct\Delta}{9}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)\left(\frac{P\alpha}{tc(c-\alpha)\Delta^2}\right) \\
&= \frac{ct}{18}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)\left(\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right) + 2\Delta\left(\frac{P\alpha}{tc(c-\alpha)\Delta^2}\right)\right) \\
&= \frac{ct}{18}\left((4-S) - \frac{P\alpha}{ct\Delta(c-\alpha)}\right)\left((4-S) + \frac{\alpha P}{tc(c-\alpha)\Delta}\right) \\
&= \frac{ct}{18}\left((4-S)^2 - \left(\frac{P\alpha}{ct\Delta(c-\alpha)}\right)^2\right).
\end{aligned}
$$

To determine the sign of marginal profit, we show that both parentheses in (17) are positive. Notice first that

$$
\frac{P}{tc(c-\alpha)\Delta} < \frac{S+2}{2\alpha}
$$

because of $q^*(x_2) \in Q_{III}$. Since $x_2 \in (1/2, 1]$ implies $S < 2$ and hence

$$
\frac{S+2}{2\alpha} < \frac{4-S}{\alpha},
$$

it follows that

$$
\frac{P}{tc(c-\alpha)\Delta} < \frac{4-S}{\alpha},
$$

i.e. the first term in parentheses in (17) is positive. Moreover, the second term in parentheses in (17) is positive because $x_2 \leq 1$ implies $4 - S - 2\Delta > 0$. Thus $\pi_2'(x_2) > 0$ for any location x_2 and any corresponding quality equilibrium of type III.

Type IV: Let $q^*(x_2) \in Q_{IV}$ and $x_2 \in int\ X_{IV}$, i.e. we have $q_1^*(x_2) = P/c$ and

$$q_2^*(x_2) = q_2^{FOC}\left(\frac{P}{c}\right) = \frac{P}{c} - \frac{t\Delta(2-S)}{2} > 0.$$

From $q_2^*(x_2) = P/c - t\Delta(2-S)/2 > 0$, we obtain a quality gap of

$$q_1^*(x_2) - q_2^*(x_2) = \frac{t\Delta(2-S)}{2} > 0$$

and a demand of

$$\begin{aligned} d_2^*(x_2) &= 1 - \frac{S}{2} - \frac{q_1^* - q_2^*}{2t\Delta} \\ &= \frac{1}{2} - \frac{S}{4}. \end{aligned}$$

We insert the above expressions to get

$$\begin{aligned} \pi_2(x_2) &= (P - cq_2^*)\, d_2^* \\ &= \left(\frac{1}{2}tc\Delta(2-S)\right)\left(\frac{1}{2} - \frac{S}{4}\right) \\ &= \frac{tc\Delta(2-S)^2}{8} > 0. \end{aligned}$$

Calculating $\pi_2'(x_2)$, we obtain

$$\begin{aligned} \frac{\partial \pi_2}{\partial x_2} &= \frac{\partial \pi_2}{\partial S} + \frac{\partial \pi_2}{\partial \Delta} \\ &= -\frac{tc\Delta(2-S)}{4} + \frac{tc(2-S)^2}{8} \\ &= \frac{1}{8}ct(2-S)(2-S-2\Delta) \end{aligned}$$

Because of $S < 2$, the first order condition $\pi_2'(x_2) = 0$ reduces to $2 - S - 2\Delta = 0$, which yields $x_2^* = 5/6$ as the unique candidate for a profit maximum. The second order condition is satisfied because of

$$\begin{aligned} \frac{\partial^2 \pi_2}{\partial x_2^2} &= \frac{\partial}{\partial x_2}\left(\frac{tc(2-S)^2}{8} - \frac{1}{4}ct\Delta(2-S)\right) \\ &= \frac{ct}{4}\left(3x_2 - \frac{7}{2}\right) < 0, \end{aligned}$$

as the inequality is true for all $x_2 \in (1/2, 1]$. Thus, an interior profit maximum exists at $x_2 = 5/6$. ■

Proposition 4 (Location choice - technical version) *Fix $P > 0$, set $\widetilde{P} = P/ (tc\,(c - \alpha))$, and let $\widehat{x}$ be given by (7). Moreover, define*

$$x^{I,IIa} = \left(\frac{P}{t\,(c-\alpha)} + \frac{1}{4}\right)^{1/2} \quad \text{and} \quad x^{IIb,III} = \frac{1}{2}\sqrt{12Pc - 4P\alpha + 9} - 1 \quad (18)$$

as the locations corresponding to the boundaries of equilibrium regions I and IIa and of IIb and III, respectively.

(a) If the price is high, $\widetilde{P} > 7/(8\alpha)$, then the private hospital locates at $x_2^* = 5/6$.

(b) Suppose the price is low, i.e. $\widetilde{P} < 5/(9\alpha)$.

(i) If altruism is strong, $\alpha/c \geq 2/3$, then we have $\widehat{x} \geq 1$ and the private hospital locates at $x_2^* = \min\left\{x^{I,IIa}, 1\right\}$. Furthermore, this location is interior, $x^{I,IIa} < 1$, if either $\alpha/c \geq 20/27$ or $\widetilde{P} < 3/(4c)$ (or both) hold true.

(ii) If altruism is weak, $\alpha/c < 2/3$, then we have $\widehat{x} < 1$ and up to three cases can occur:[11]
For $\widetilde{P} \leq \frac{2\alpha}{(2c-\alpha)^2}$, the boundary location $\widehat{x}$ corresponds to equilibrium region I, $\widehat{x} \in X_I$. In this case, the private hospital locates at $x_2^* = x^{I,IIa} \leq \widehat{x}$.
For $\widetilde{P} \in \left(\frac{2\alpha}{(2c-\alpha)^2}, \min\left\{\frac{5}{9\alpha}, \frac{7}{4tc(c-\alpha)(3c-\alpha)}\right\}\right)$, we have $\widehat{x} \in X_{III} \cup X_{IV}$, and the private hospital locates at $x_2^* = x^{IIb,III} \in [\widehat{x}, 1)$.
For $\widetilde{P} \in \left(\max\left\{\frac{2\alpha}{(2c-\alpha)^2}, \frac{7}{4tc(c-\alpha)(3c-\alpha)}\right\}, \frac{5}{9\alpha}\right)$, we have $\widehat{x} \in X_{III} \cup X_{IV}$, and the private hospital locates at the corner, i.e. $x_2^* = 1$.

Proof of Proposition 4. Let $P > 0$ and $x_2 \in (1/2, 1]$ be given arbitrarily and let $q^* = (q_1^*(x_2), q_2^*(x_2))$ denote the corresponding quality equilibrium that results at stage 3. Correspondingly, let $\pi_2(x_2) = \pi_2(q_1^*(x_2), q_2^*(x_2))$ denote the reduced profit function of the private hospital at stage 2. The private hospital chooses x_2 in order to maximize profit, $\pi_2(x_2) = (P - cq_2^*(x_2))\, d_2^*(x_2)$, where equilibrium demand $d_2^*(x_2)$ is given by (16).

Part (a): Suppose the price is high, i.e. $\widetilde{P} = P/(tc\,(c-\alpha)) > 7/(8\alpha)$. In this case $x_2 \in (1/2, 1]$ implies $\Delta(S+2)/(2\alpha) \leq 7/(8\alpha)$. It hence follows from Proposition 2 that any quality equilibrium q^* is of type IV. Therefore the claim follows from Proposition 3.

Part (b): Suppose the price is low, i.e. $\widetilde{P} = P/(tc\,(c-\alpha)) < 5/(9\alpha)$. In this case, location $x_2 = 5/6$ does not belong to equilibrium region IV because of $[\Delta(S+2)/(2\alpha)]_{x_2=5/6} = 5/(9\alpha)$. By Corollary 1, this implies that $x_2 < 5/6$ for all $x_2 \in X_{IV}$. It hence follows from Proposition 3 that $\partial\pi_2/\partial x_2 > 0$ for all $x_2 \in X_{IV}$.

[11] Depending on parameter values the second or third interval can be empty.

Case (i): Let $\alpha/c > 2/3$ and observe that this implies $\widehat{x} > 1$. It hence follows from part (a) of Proposition 2 that any location $x_2 \in (1/2, 1]$ belongs to equilibrium region I, IIa, III, or IV (and none to IIb). By Proposition 3, we have $\partial\pi_2/\partial x_2 > 0$ for all $x_2 \in X_{IV} \cup X_{III} \cup X_{IIa}$ and $\partial\pi_2/\partial x_2 < 0$ for all $x_2 \in X_I$. Therefore, the only remaining local maximum is at $x_2^* = \min\left\{x^{I,IIa}, 1\right\}$, which then must be global.

Moreover, this location is interior, $x^{I,IIa} < 1$, if and only if $\widetilde{P} < 3/(4c)$. If $\alpha/c \geq 20/27$, the latter condition is always satisfied because of $\widetilde{P} < 5/(9\alpha) \leq 3/(4c)$. If, on the other hand, $\alpha/c \in (2/3, 20/27)$, then we have $x_2^* = x^{I,IIa} < 1$ for $\widetilde{P} < 3/(4c)$ and $x_2^* = 1$ for $\widetilde{P} \in [3/(4c), 5/(9\alpha))$.

Case (ii): Now consider $\alpha/c \leq 2/3$. In this case, we have $\widehat{x} \leq 1$, i.e., by Proposition 2, no $x_2 < \widehat{x}$ belongs to equilibrium region IIb and no $x_2 > \widehat{x}$ to region IIa.

First, notice that $\widetilde{P} \leq 2\alpha/(2c-\alpha)^2$ if and only if $\widehat{x} \in X_I$. In this case, any $x_2 > \widehat{x}$ implies $x_2 \in X_I$ by Corollary 1, i.e. equilibrium region IIb is empty. Hence, Proposition 3 implies $x_2^* = x^{I,IIa} \leq \widehat{x}$.

Second, if $\widetilde{P} > 2\alpha/(2c-\alpha)^2$ then it follows from part (c) of Proposition 2 that $\widehat{x} \in X_{III} \cup X_{IV}$ and hence $x_2 \in X_{III} \cup X_{IV}$ for all $x_2 < \widehat{x}$. Therefore, Proposition 3 implies that $\partial\pi_2/\partial x_2 > 0$ for all $x_2 \leq \widehat{x}$ and that $\partial\pi_2/\partial x_2 < 0$ for all $x_2 \in X_{IIb} \cup X_I$. Thus, $x_2^* = \min\left\{x^{IIb,III}, 1\right\}$.

Observe that this location is interior, $x^{IIb,III} < 1$, if and only if

$$\widetilde{P} < 7/(4tc(c-\alpha)(3c-\alpha)),$$

i.e. we have $x_2^* = x^{IIb,III} \in [\widehat{x}, 1)$ for $\widetilde{P} < \min\{5/(9\alpha), 7/(4tc(c-\alpha)(3c-\alpha))\}$ and $x_2^* = 1$ for $\widetilde{P} \in [7/(4tc(c-\alpha)(3c-\alpha)), 5/(9\alpha))$. ■

Proposition 5 (Technical version) *Set $\widetilde{B} = B/(tc(c-\alpha))$ and suppose that the budget is small, $\widetilde{B} < 5/(9\alpha)$, and altruism is strong, $\alpha/c \geq 1/8$. Then we have:*
(a) If $\widetilde{B} \leq \frac{4}{9c}$ then $P^* = B$ maximizes (8) subject to $P \leq B$.
(b) If $\widetilde{B} \in \left(\frac{4}{9c}, \frac{5}{9\alpha}\right)$ then $P^* = \frac{4}{9}t(c-\alpha)$ maximizes (8) subject to $P \leq B$; the regulator does not spend the full budget, i.e. $P^* < B$.
In both cases, we have $q_1^* = q_2^* = 0$ and the provider locates the further to the right the higher the regulator sets the price. The constrained welfare optimum is realized when transportation costs are minimized, i.e. in case (b).

Proof of Proposition 5 Consider $\widetilde{B} < 5/(9\alpha)$ and $\alpha/c \geq 1/8$. The budget constraint $P \leq B$ implies that the price is low. Hence, part (b) of Proposition 4 implies $x_2^* = x^{I,IIa} < 1$ and $(q_1^*, q_2^*) = (0, 0)$ for any $P \leq B$. If $\alpha/c \geq 2/3$ this follows from Proposition 4(b), part (i). For $\alpha/c < 2/3$ it follows from Proposition 4(b), part (ii), since $\alpha/c \geq 1/8$ implies

$$\frac{4}{9c} \leq \frac{2\alpha}{(2\alpha - c)^2}.$$

Moreover, by equation (18), the location $x_2^* = x^{I,IIa}$ monotonically increases in P.

Because of $(q_1^*, q_2^*) = (0,0)$, maximizing (8) is equivalent to minimizing transportation cost. As argued in section 4, transportation cost is decreasing (increasing) for $x_2 < (>) \, 5/6$ so that $x_2 = 5/6$ represents the welfare optimal location (budget permitting). In order to implement $x_2 = 5/6$, the regulator solves $x^{I,IIa} = 5/6$ for P to obtain $P^* = 4t\,(c-\alpha)\,/9$. Accordingly, if $\widetilde{B} \geq 4/\,(9c)$ the regulator sets $P = P^*$ and if $\widetilde{B} < 4/\,(9c)$ then spending the full budget $P^* = B$ implements the location

$$x^{I,IIa} = \left(\frac{B}{t\,(c-\alpha)} + \frac{1}{4}\right)^{1/2} < \frac{5}{6},$$

which maximizes social welfare subject to $P \leq B$, since welfare is increasing in x_2 and hence in P. ■

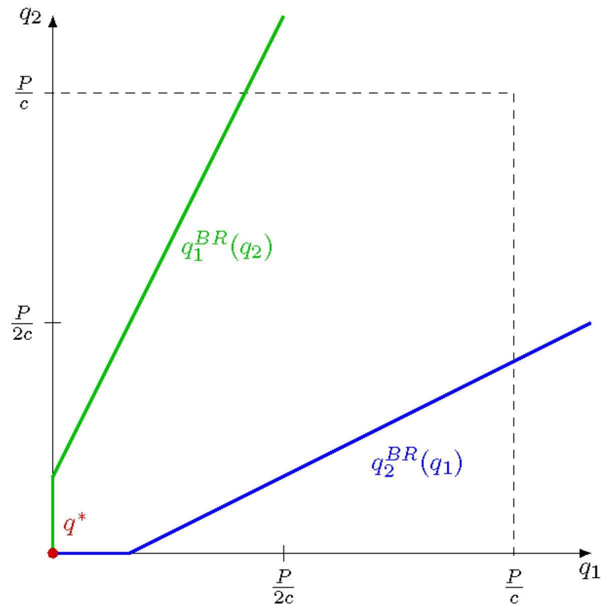

Figure 1(a): Quality equilibrium of type I

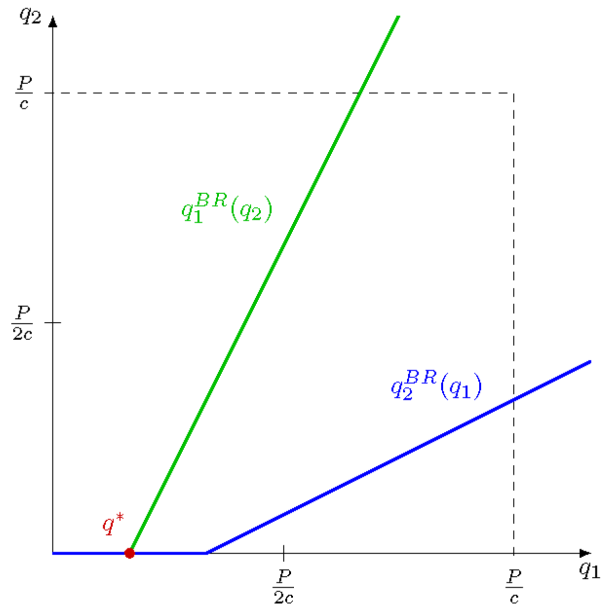

Figure 1(b): Quality equilibrium of type IIa

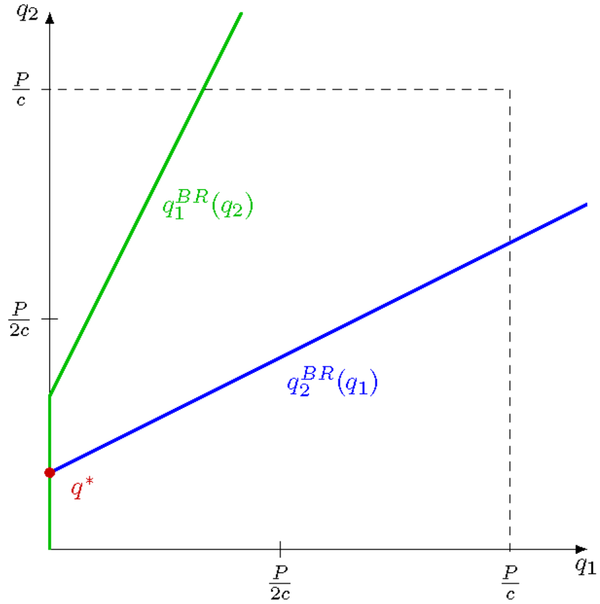

Figure 1(c): Quality equilibrium of type IIb

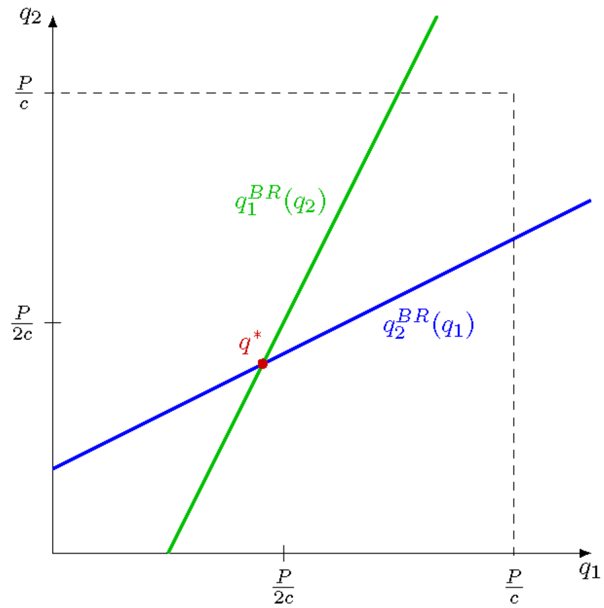

Figure 1(d): Quality equilibrium of type III

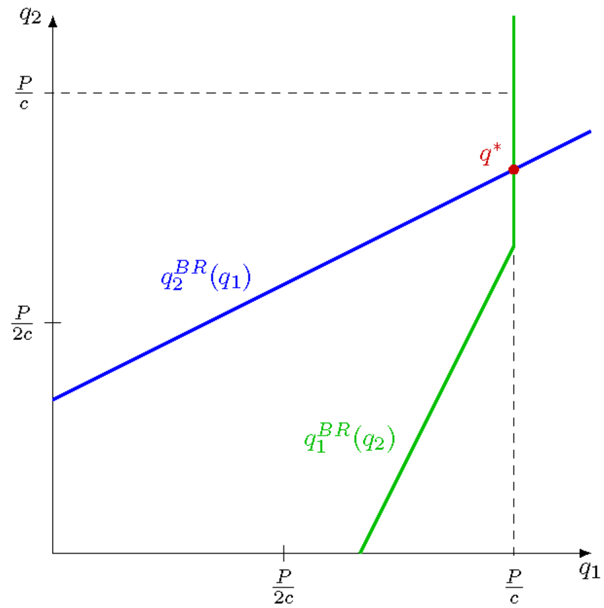

Figure 1(e): Quality equilibrium of type IV

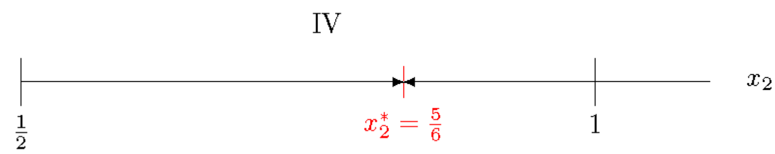

Figure 2(a): Location choice for high prices

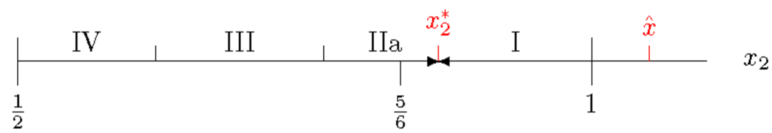

Figure 2(b): Location choice for a low price and strong altruism ($\frac{\alpha}{c} \geq \frac{2}{3}$)

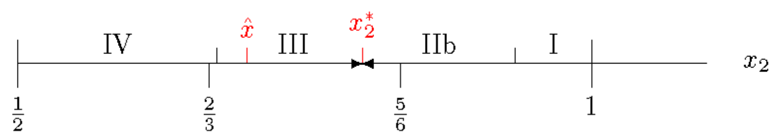

Figure 2(c): Location choice for a moderately low price and weak altruism ($\frac{\alpha}{c} < \frac{2}{3}$)